A job fair is taking place in an auditorium which can hold a maximum of 640 jobseekers. **SCORE: _____ / 18 PTS**

At noon, there were 160 jobseekers in the auditorium, of whom 5% were currently unemployed. (The rest were looking to switch jobs.)
Every hour, 300 additional jobseekers entered the auditorium, of whom 10% were currently unemployed.
At the same time, 180 jobseekers left the auditorium every hour.
(Assume that these jobseekers leaving were perfectly representative of all the jobseekers in the auditorium at the same time. That is, at any time, the percent of currently unemployed people in the two groups were equal to each other.)

Find the number of unemployed jobseekers in the auditorium at the moment when the maximum capacity was reached.
NOTE: Although the number of unemployed jobseekers is not a continuous function, it can be modelled by one.
HINT: Simplify all fractions as soon as possible.

$$\frac{dA}{dt} = 10\% * 300 - \frac{A}{160+(300-180)t} * 180$$

MAXIMUM CAPACITY WHEN $160+120t=640$

$t=4$

$$= 30 - \frac{180A}{160+120t} \quad 3$$

$$= 30 - \frac{9A}{8+6t}, \quad A(0) = 5\% * 160 = 8$$

$$\frac{dA}{dt} + \frac{9}{8+6t}A = 30$$

$$\mu = e^{\int \frac{9}{8+6t}dt} = e^{\frac{9}{6}\ln|8+6t|} = (8+6t)^{\frac{3}{2}}$$

$$(8+6t)^{\frac{3}{2}}\frac{dA}{dt} + 9(8+6t)^{\frac{1}{2}}A = 30(8+6t)^{\frac{3}{2}}$$

CHECKPOINT:

$$\frac{d}{dt}(8+6t)^{\frac{3}{2}} = \frac{3}{2}\cdot 6(8+6t)^{\frac{1}{2}} = 9(8+6t)^{\frac{1}{2}}$$

✓

$$(8+6t)^{\frac{3}{2}}A = \int 30(8+6t)^{\frac{3}{2}}dt + C$$

$$= 30\cdot\frac{2}{5}\cdot\frac{1}{6}(8+6t)^{\frac{5}{2}} + C$$

$$= 2(8+6t)^{\frac{5}{2}} + C$$

$$A = 2(8+6t) + C(8+6t)^{-\frac{3}{2}}$$

$$8 = 2(8) + C(8)^{-\frac{3}{2}}$$

$$8 = 16 + 8^{-\frac{3}{2}}C$$

$$2 \quad C = -8^{\frac{5}{2}}$$

$$A = 2(8+6t) - 8^{\frac{5}{2}}(8+6t)^{-\frac{3}{2}}$$

$$A(4) = 2(32) - 8^{\frac{5}{2}}(32)^{-\frac{3}{2}}$$

$$= 64 - (2^3)^{\frac{5}{2}}(2^5)^{-\frac{3}{2}}$$

$$= 64 - 1$$

$$2 \quad = 63$$

In class, we discussed the logistic population model

SCORE: _____ / 12 PTS

ON BOTH QUESTIONS, ALL ITEMS WORTH (1) POINT UNLESS OTHERWISE INDICATED

$$\frac{dP}{dt} = -kP(P-a), \quad P(0) = P_0 \text{ (where } k \text{ and } a \text{ are positive constants)}$$

Solve the generalized version of this model

$$\frac{dP}{dt} = -kP(P^n-a), \quad P(0) = P_0 \text{ (where } k, n \text{ and } a \text{ are positive constants)}$$

NOTE: Your final answer must be an explicit function.

$$\frac{dP}{dt} - kaP = -kP^{n+1}$$

$$v = P^{1-(n+1)} = P^{-n}$$

$$\frac{dv}{dt} = -nP^{-n-1}\frac{dP}{dt}$$

$$\frac{dP}{dt} = -\frac{1}{n}P^{n+1}\frac{dv}{dt}$$

$$-\frac{1}{n}P^{n+1}\frac{dv}{dt} - kaP = -kP^{n+1}$$

$$\frac{dv}{dt} + nkaP^{-n} = nk$$

$$\frac{dv}{dt} + nkav = nk$$

$$\mu = e^{\int nka\,dt} = e^{nkat}$$

$$e^{nkat}\frac{dv}{dt} + nkae^{nkat}v = nke^{nkat}$$

CHECKPOINT: $\frac{d}{dt}e^{nkat} = nkae^{nkat}$ ✓

$$e^{nkat}v = \int nke^{nkat}\,dt + C$$

$$= \frac{1}{a}e^{nkat} + C$$

$$P^{-n} = v = \frac{1}{a} + Ce^{-nkat}$$

$$P = \left(\frac{1}{a} + Ce^{-nkat}\right)^{-\frac{1}{n}}$$

$$P_0 = \left(\frac{1}{a} + C\right)^{-\frac{1}{n}}$$

$$C = P_0^{-n} - \frac{1}{a}$$

$$P(t) = \left(\frac{1}{a} + \left(P_0^{-n} - \frac{1}{a}\right)e^{-nkat}\right)^{-\frac{1}{n}}$$